■ Adomian polynomials

The entire paper

Abdul-Majid Wazwaz, “A new algorithm for calculating Adomian polynomials for nonlinear operators”, *Applied Mathematics and Computation* **111** (2000) 53-69

can be reproduced in a trivial fashion. After defining the perturbation expansion, trunctated after *n* terms,

In[1]:= $\Phi_{n\_} := \sum_{i=0}^{n-1} u_i\,\alpha^i + O[\alpha]^n$

and its derivative,

In[2]:= $\Phi^{(m\_)}{}_{n\_} := \sum_{i=0}^{n-1} u_i{}'\,\alpha^i + O[\alpha]^n$

here are the expansions in Warwaz (2000). The Adomian polynomial A_i are the coefficients of α^i in each case.

In[3]:= $\Phi_5\,\Phi'{}_5$

Out[3]= $u_0\,u_0{}' + (u_1\,u_0{}' + u_0\,u_1{}')\,\alpha + (u_2\,u_0{}' + u_1\,u_1{}' + u_0\,u_2{}')\,\alpha^2 + (u_3\,u_0{}' + u_2\,u_1{}' + u_1\,u_2{}' + u_0\,u_3{}')\,\alpha^3 + (u_4\,u_0{}' + u_3\,u_1{}' + u_2\,u_2{}' + u_1\,u_3{}' + u_0\,u_4{}')\,\alpha^4 + O(\alpha^5)$

In[4]:= $\Phi_4{}^{\tau}$

Out[4]= $u_0^{\tau} + \tau\,u_0^{\tau-1}\,u_1\,\alpha + \left(\frac{1}{2}\,(\tau-1)\,\tau\,u_1^2\,u_0^{\tau-2} + \tau\,u_2\,u_0^{\tau-1}\right)\alpha^2 + \left(\frac{1}{6}\,(\tau-2)\,(\tau-1)\,\tau\,u_1^3\,u_0^{\tau-3} + (\tau-1)\,\tau\,u_1\,u_2\,u_0^{\tau-2} + \tau\,u_3\,u_0^{\tau-1}\right)\alpha^3 + O(\alpha^4)$

In[5]:= $\Phi_4{}^{3/2}$ // Simplify

Out[5]= $u_0^{3/2} + \frac{3}{2}\,\sqrt{u_0}\,u_1\,\alpha + \frac{3\,(u_1^2 + 4\,u_0\,u_2)\,\alpha^2}{8\,\sqrt{u_0}} + \frac{(-u_1^3 + 12\,u_0\,u_2\,u_1 + 24\,u_0^2\,u_3)\,\alpha^3}{16\,u_0^{3/2}} + O(\alpha^4)$

In[6]:= $\Phi_5{}^2$

Out[6]= $u_0^2 + 2\,u_0\,u_1\,\alpha + (u_1^2 + 2\,u_0\,u_2)\,\alpha^2 + (2\,u_1\,u_2 + 2\,u_0\,u_3)\,\alpha^3 + (u_2^2 + 2\,u_1\,u_3 + 2\,u_0\,u_4)\,\alpha^4 + O(\alpha^5)$

In[7]:= $\Phi_5{}^3$ // Simplify

Out[7]= $u_0^3 + 3\,u_0^2\,u_1\,\alpha + 3\,u_0\,(u_1^2 + u_0\,u_2)\,\alpha^2 + (u_1^3 + 6\,u_0\,u_2\,u_1 + 3\,u_0^2\,u_3)\,\alpha^3 + 3\,(u_2\,u_1^2 + 2\,u_0\,u_3\,u_1 + u_0\,(u_2^2 + u_0\,u_4))\,\alpha^4 + O(\alpha^5)$

In[8]:= Factor /@ $(\Phi_5^2\,\Phi'{}_5)$

Out[8]= $u_0^2\,u_0{}' + u_0\,(2\,u_1\,u_0{}' + u_0\,u_1{}')\,\alpha + (u_2{}'\,u_0^2 + 2\,u_2\,u_0{}'\,u_0 + 2\,u_1\,u_1{}'\,u_0 + u_1^2\,u_0{}')\,\alpha^2 + (u_3{}'\,u_0^2 + 2\,u_3\,u_0{}'\,u_0 + 2\,u_2\,u_1{}'\,u_0 + 2\,u_1\,u_2{}'\,u_0 + 2\,u_1\,u_2\,u_0{}' + u_1^2\,u_1{}')\,\alpha^3 + (u_4{}'\,u_0^2 + 2\,u_4\,u_0{}'\,u_0 + 2\,u_3\,u_1{}'\,u_0 + 2\,u_2\,u_2{}'\,u_0 + 2\,u_1\,u_3{}'\,u_0 + u_2^2\,u_0{}' + 2\,u_1\,u_3\,u_0{}' + 2\,u_1\,u_2\,u_1{}' + u_1^2\,u_2{}')\,\alpha^4 + O(\alpha^5)$

In[9]:= Factor /@ $\sin(\Phi_5)$

Out[9]= $\sin(u_0) + \cos(u_0)\,u_1\,\alpha + \frac{1}{2}\,(2\cos(u_0)\,u_2 - \sin(u_0)\,u_1^2)\,\alpha^2 + \frac{1}{6}\,(-\cos(u_0)\,u_1^3 - 6\sin(u_0)\,u_2\,u_1 + 6\cos(u_0)\,u_3)\,\alpha^3 + \frac{1}{24}\,(\sin(u_0)\,u_1^4 - 12\cos(u_0)\,u_2\,u_1^2 - 24\sin(u_0)\,u_3\,u_1 - 12\sin(u_0)\,u_2^2 + 24\cos(u_0)\,u_4)\,\alpha^4 + O(\alpha^5)$

`In[10]:=` **Factor /@ exp(Φ₅)**

`Out[10]=` $$e^{u_0} + e^{u_0}\, u_1\, \alpha + \frac{1}{2} e^{u_0}\, (u_1^2 + 2\, u_2)\, \alpha^2 + \frac{1}{6} e^{u_0}\, (u_1^3 + 6\, u_2\, u_1 + 6\, u_3)\, \alpha^3 + \frac{1}{24} e^{u_0}\, (u_1^4 + 12\, u_2\, u_1^2 + 24\, u_3\, u_1 + 12\, u_2^2 + 24\, u_4)\, \alpha^4 + O(\alpha^5)$$

`In[11]:=` **ln(Φ₅) // ExpandAll**

`Out[11]=` $$\log(u_0) + \frac{u_1\, \alpha}{u_0} + \left(\frac{u_2}{u_0} - \frac{u_1^2}{2\, u_0^2}\right)\alpha^2 + \left(\frac{u_1^3}{3\, u_0^3} - \frac{u_2\, u_1}{u_0^2} + \frac{u_3}{u_0}\right)\alpha^3 + \left(-\frac{u_1^4}{4\, u_0^4} + \frac{u_2\, u_1^2}{u_0^3} - \frac{u_3\, u_1}{u_0^2} - \frac{u_2^2}{2\, u_0^2} + \frac{u_4}{u_0}\right)\alpha^4 + O(\alpha^5)$$