ElectroMagnetism — Assignment 1

Solutions are to be handed in at 9am on Monday August 24.

1. Consider the potential $\phi(x, y) = x\left(x^2 + y^2 + 2\,y\right)$.

(a) Visualize $\phi(x, y)$. [2]

(b) Compute the corresponding electric field and determine its value at $(1, 0)$. In what direction is the potential decreasing most rapidly at this point? [2]

(c) Find the *critical point*(*s*) (those (x, y) for which $\mathcal{E}_x = -\frac{\partial\phi(x,y)}{\partial x} = 0$ and $\mathcal{E}_y = -\frac{\partial\phi(x,y)}{\partial y} = 0$). [2]

(d) What is the potential at the *critical point*(*s*)? [2]

(e) Classify the *critical point*(*s*) as maxima, minima, or saddle-points. What can you say about the stability of a test charge placed at each critical point? [2]

2. Let $\boldsymbol{r}$ be the vector from a fixed point (a, b, c) to (x, y, z) and let $r = \|\boldsymbol{r}\|$ be its length.

(a) Compute $\nabla\left(\frac{1}{r}\right)$. [2]

(b) Compute $\nabla(f(r))$ where f is an arbitrary scalar function. [3]

3. A sphere of radius R centred at the origin has charge density

$$\rho(r, \theta) = r\cos(\theta),$$

where r, θ, are spherical coordinates.

To find the approximate potential for points on the z-axis, far from the sphere proceed as follows:

(a) Compute the total charge (*monopole term*). [3]

(b) Compute the *dipole term*. [4]

4. The field around a charge q in a plasma has the form

$$\boldsymbol{E} = e^{-r/\lambda}\,\frac{q}{4\,\pi\,\epsilon_0\,r^2}\,\hat{\boldsymbol{r}},$$

where λ is a constant.

(a) Calculate the divergence of this field. [2]

(b) Determine the charge density using Gauss' Law. [2]

(c) Compute the charge, $Q(R) = \int_{V(R)} \rho(\boldsymbol{r})\,d\boldsymbol{r}$, enclosed in a sphere of radius R centred on q. [3]

(d) Compute the total flux, $\Phi(R) = \oint_{S=\partial V(R)} \boldsymbol{E}\cdot d\boldsymbol{S}$, emergent from a shell of radius R. Compare the result with your answer to (c). [3]

(e) Interpret $Q(R)$ physically. [2]